Lessons 5-1 & 5-2 Extra Practice

AP Calculus AB
Lesson 5-1 & 5-2 Extra Practice

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Date

1. Find the absolute extreme values of the function $f(x) = 2x^3 - 3x^2 - 12x$ on the interval $[0,3]$.

$f'(x) = 6x^2 - 6x - 12$

$0 = 6x^2 - 6x - 12$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$\boxed{x = 2}$

just this value is found in the interval $[0,3]$.

$f(0) = 0$ Absolute max (largest value)

$f(2) = -20$ Absolute min (smallest value)

$f(3) = -9$

Absolute max at $(0,0)$ and Absolute min at $(2,-20)$

2. Find the absolute extreme values of $f(x) = \begin{cases} 5 - 2x^2, & x \le 1 \\ x + 2, & x > 1 \end{cases}$ on the interval $\left[-\frac{1}{2}, 4\right]$.

$f'(x) = -4x$

$f'(x) = 1$

Critical values

$-4x = 0$ $\quad f'(1) = -4$

① $x = 0$ $\quad f'(1) = 1$ $\Big\} \neq$

② $x = 1$

$f(-\frac{1}{2}) = 5 - 2(-\frac{1}{2})^2 = 5 - \frac{1}{2} = 4.5$

$f(0) = 5$

$f(1) = 3$ Min

$f(4) = 6$ Max

Absolute max at $(4, 6)$

Absolute min at $(1, 3)$

OVER →

Lessons 5-1 & 5-2 Extra Practice

1. $F(x) = \frac{3}{3}x^3 - \frac{2}{2}x^2 + 3x + C$

$= x^3 - x^2 + 3x + C$

$5 = 1^3 - 1^2 + 3 \cdot 1 + C$

$5 = 3 + C$

$2 = C$

$\boxed{F(x) = x^3 - x^2 + 3x + 2}$

2. $F(x) = -2\cos(\frac{1}{2}x) + C$

$0 = -2\cos(\frac{1}{2} \cdot \frac{\pi}{2}) + C$

$0 = -2\cos\frac{\pi}{4} + C$

$0 = \cancel{-2} \cdot \frac{\sqrt{2}}{\cancel{2}} + C$

$0 = -\sqrt{2} + C$

$C = \sqrt{2}$

$\boxed{F(x) = -2\cos(\frac{1}{2}x) + \sqrt{2}}$

3. $f'(x) = 6x^2 - 6$

* Slope of T.L.'s

parallel means slopes are =

$6x^2 - 6 = 0$

$6(x^2 - 1) = 0$

$6(x+1)(x-1) = 0$

$x = \pm 1$

$x = 1$ falls on the interval

$f(1) = 6(1)^2 - 6$

$= 0$

$\boxed{\text{The point } (1,0)}$

Slope of secant line $= \frac{f\sqrt{3} - f(0)}{\sqrt{3} - 0}$

$= \frac{[2(\sqrt{3})^3 - 6\sqrt{3}] - 0}{\sqrt{3}}$

$= \frac{2(\sqrt{3})^3 - 6\sqrt{3}}{\sqrt{3}}$

$= \frac{\cancel{\sqrt{3}}(2(\sqrt{3})^2 - 6)}{\cancel{\sqrt{3}}}$

$= 2 \cdot 3 - 6$

$= 0$

Lessons 5-1 & 5-2 Extra Practice

Do all work on a separate sheet of paper!

Practice
No Calculator

1. Find the antiderivative of $f'(x)=3x^2-2x+3$ if $f(1)=5$

2. Find the antiderivative of $f'(x)=\sin\left(\frac{1}{2}x\right)$ if $f\left(\frac{\pi}{2}\right)=0$

3. If $f(x)=2x^3-6x$, at what point on the interval $0\le x\le\sqrt{3}$ (if any) is the tangent line to the curve parallel to the secant line on that interval?

2016 AP Exam
No calculator:

25. Let f be a function with first derivative defined by $f'(x)=\frac{3x^2-6}{x^2}$ for $x>0$. It is known that $f(1)=9$ and $f(3)=11$. What value of x in the open interval $(1,3)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1,3]$?

MVT guarantees there is a # x with $1<x<3$ satisfying

$$f'(x)=\frac{f(3)-f(1)}{3-1}=\frac{11-9}{2-1}=\frac{2}{2}=1$$

(A) $\sqrt{6}$ (B) $\sqrt{3}$ (C) $\sqrt{2}$ (D) 1

$$\frac{3x^2-6}{x^2}=1$$
$$3x^2-6=x^2$$
$$2x^2-6=0$$
$$x^2-3=0$$
$$x=\pm\sqrt{3}$$ (choose $\sqrt{3}$)

29. The function f is defined by $f(x)=x^3+4x+2$. If g is the inverse function of f and $g(2)=0$, what is the value of $g'(2)$?

$f'(x)=3x^2+4$

$f^{-1}: (2,0)$

$f: (0,2)$

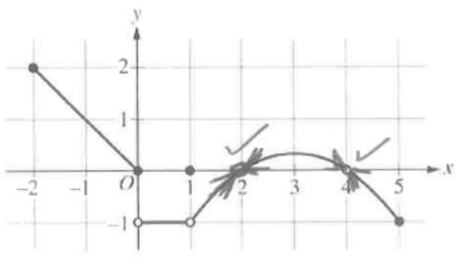
(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4

$$g'(2)=\left.\frac{df^{-1}}{dx}\right|_{x=2}=\frac{1}{\left.\frac{df}{dx}\right|_{x=0}}=\frac{1}{3(0)^2+4}=\frac{1}{4}$$

77. The graph of the function f is shown above. For what values of a does $\lim_{x\to a} f(x)=0$?

LS=0 AND
RS=0

(A) ~~2 only~~
(B) 2 and 4
(C) 0 and 2 only
(D) 0, 1, and 2



Graph of f